Original Research Article

The Product Life Cycle of Durable Goods

Abstract

A dynamic model of the product lifecycle of nearly homogeneous durables in polypoly markets is established. It is based on the idea that the market dynamics is governed mainly by two processes: the spreading of a durable into the market and the demand and supply specifying the mean price of the good. Taking advantage from the Bass model for first purchase and a logistic model describing the repurchase process the entire product lifecycle of durable goods can be established in agreement with empirical data studied in this paper. The decline phase of the lifecycle is modelled here as the occurrence of a negative repurchase rate. For the case of a fast growing supply it is shown that the mean price of the good decreases according to a logistic law.

The presented approach discusses the interference of the diffusion process with the supply dynamics. The theory predicts the occurrence of lost sales in the initial stages of the lifecycle due to supply constraints. They are the origin for a delayed market penetration. The imitation rate B indicating social contagion decreases with decreasing amounts of available units at introduction and a slow output increase in time. A comparison with empirical data indicates the qualitative agreement of this relationship.

Keywords: Product Lifecycle, Consumer Durables, Product Diffusion, Bass Diffusion, Logistic Growth, Supply Constraints

1. Introduction

A microeconomic model is presented that merges the product lifecycle (PLC) concept of durables with the price evolution of homogeneous goods in polypoly markets. The idea of a product lifecycle suggests that analogous to the life of organisms goods are subject to characteristic stages in their evolution separated usually into introduction, growth, maturity and decline phase [1]. They can be separated by the dominant growth process governing the market penetration. The initial stages of the lifecycle describe the diffusion of a good into a market. This process was studied intensively in order to forecast the adopter and sales evolution of durable goods [2-4]. The PLC concept finds not only application as a forecasting tool and guideline for a corporate marketing strategy but also in operation research in order to specify the size of production capacities [5-7]. In order to establish the entire PLC next to market diffusion also repurchase has to be taken into account. The latter must be proportional to the number of previous adopters a repurchase rate. Here we take advantage of a logistic growth model of the repurchase rate similar to non-durable goods [8].

The price is a key variable in microeconomic considerations [9]. The lifecycle concept incorporates the spreading of a good into the market while the price evolution is a consequence of the relation between supply and demand. In order to combine both concepts the market dynamics of a polypoly market is considered to be governed by the meeting of demanded (required) and supplied (available) product units. While the spreading process determines the number of demanded units, the number of available units is a consequence of the output flow of suppliers. The implementation of the price as a decision variable in the spreading process of durables turned out to be difficult. It is usually taken into account as a perturbation variable in the diffusion process [4,10,11]. In order the price evolution into account a dynamic model explaining the price dispersion of homogeneous goods is applied [12,13]. It suggests that if the purchase process can be understood as the meeting of demanded and supplied units the price dispersion of a homogenous good has for a short time period the form of a Laplace distribution. The mean price is determined by a Walrus equation suggesting an increase of the price for an excess growth of demanded units and a decrease for the case of an excess growth of supplied units [14]. The presented model establishes a relation between market penetration and the supply evolution of durable markets.

The paper is organized as follows. The next section is devoted to a presentation of the model of the PLC of durable goods in polypoly markets. It starts with the derivation of the market dynamics of durables followed by a consideration of the price dynamics of homogenous goods. After establishing the PLC of durables the impact of the supply side on the rate of adoption is studied. The presented approach is applied to empirical investigations of the PLC of nearly homogenous consumer durables in the third section of the paper. The theory of the PLC of durable goods is summarized in the discussion section, followed by the main conclusions of this work.

2. The Model

The presented model is established for a polypoly market of durable goods that can be characterized as (nearly) homogeneous.[1]

2.1. The Dynamics of Polypoly Markets

The demand side of a durable market can be specified by the total number of demanded (desired) units at time step t generated by potential buyers, denoted $\tilde{x}(t)$. The supply side on the other hand is determined by the total number of supplied (available) units $\tilde{z}(t)$ offered by $N(t)>>1$ suppliers (retailers) in a polypoly market.

The total number of purchase events per unit time (total unit sales) is indicated $\tilde{y}(t)$. The key idea of the presented microeconomic approach is to consider purchase events as the meeting of demanded and supplied product units. Therefore $\tilde{y}(t)$ must disappear if the number of demanded units $\tilde{x}(t)$ or supplied units $\tilde{z}(t)$ disappears. Hence the total unit sales can be written up to the first order as a product of both variables [15]:

$$\tilde{y}(t) \cong \eta \tilde{z}(t) \tilde{x}(t) \tag{1}$$

where the unknown rate η characterizes the mean frequency by which the meeting of demanded and supplied product units generates successful purchase events. Since $\tilde{x}(t)$, $\tilde{z}(t)$, $\tilde{y}(t) \geq 0$, we demand that also $\eta \geq 0$. The evolution of the number of demanded and supplied units can be written as conservation relations of the form[2]:

[1] This is the case when the price dispersion is a (nearly) symmetric Laplacian around the mean price. Not considered are durables consisting of distinguished technological generations. If the price dispersion consists of two or more peaks the good has to be separated into different segments of homogeneous goods.
[2] In order to establish a continuous model integer variables are scaled by a large constant figure such that they can be treated as small real numbers. We demand that this scaling generates numbers $\tilde{x}(t)$, $\tilde{z}(t) \leq 1$.

$$\frac{d\tilde{x}(t)}{dt} = \tilde{d}(t) - \tilde{y}(t)$$

(2)

and

$$\frac{d\tilde{z}(t)}{dt} = \tilde{s}(t) - \tilde{y}(t)$$

(3)

Eq.(2) suggests that the total number of demanded units increases with the total demand rate $\tilde{d}(t)$ which represents the generation rate of demanded units by potential buyers. The number $\tilde{x}(t)$ decreases in time by the purchase of product units with the total unit sales rate $\tilde{y}(t)$. Eq.(3) states that the total number of supplied units increases by the supply of product units with the total supply rate (total output) $\tilde{s}(t)$ and decreases by the purchase process with the total unit sales rate $\tilde{y}(t)$. The variables in Eq.(3) can be obtained in a polypoly market from a sum over the suppliers *N(t)* by:

$$\tilde{z}(t) = \sum_{i=1}^{N(t)} z_i(t); \qquad \tilde{s}(t) = \sum_{i=1}^{N(t)} s_i(t); \qquad \tilde{y}(t) = \sum_{i=1}^{N(t)} y_i(t)$$

(4)

2.1.1. The Demand and Supply Evolution

In order to establish the market evolution we have to specify the dynamics of supply and demand. The demand side dynamics are given by Eq.(2). We want to take into account the process that demanded units do not last forever ones they are generated. Instead they have a finite mean lifetime Θ. That means, demanded units not leading to purchase events during a time period Θ disappear. This effect can be included in the demand rate $\tilde{d}(t)$ by writing:

$$\tilde{d}(t) = \tilde{d}_0(t) - \frac{\tilde{x}(t)}{\Theta}$$

(5)

where $\tilde{d}_0(t)$ describes the generation rate of demanded units by potential buyers and the second term takes the disappearance of demanded units with the rate $1/\Theta$ into account. The amount of demanded units generated by $\tilde{d}_0(t)$ can be given by:

$$\tilde{x}_0(t) = \Theta \tilde{d}_0(t)$$

(6)

The supply side is governed by the reproduction process. In a free market suppliers sell product units in order to make profit. Reinvesting a part of the profit and external money they can increase the total output $\tilde{s}(t)$ in time. This growth process can be characterized by the variable *γ(t)*, defining the relation between total supply flow and total unit sales:[3]

[3] This variable is also called reproduction parameter, since it characterizes the growth process of the output in the reproduction process.

$$\gamma(t) = \frac{\tilde{s}(t)}{\tilde{y}(t)} - 1$$

(7)

With this relation Eq.(3) can be rewritten as[4]:

$$\frac{d\tilde{z}(t)}{dt} = \gamma(t)\tilde{y}(t)$$

(8)

We want to confine here to fast growing markets with *γ(t)*>0, where the total number of supplied units provided per unit time increases the total number of sold units considerably. For a sufficiently high *γ* the number of supplied units evolves much faster than the number of demanded units in the considered time interval *Δt* such that:

$$\frac{d\tilde{x}(t)}{dt} << \frac{d\tilde{z}(t)}{dt}$$

(9)

In this case we can approximate:[5]

$$d\tilde{x}(t)/dt \cong 0$$

(10)

and obtain immediately from Eq.(2) and Eq.(5):

$$\tilde{y}(t) \cong \tilde{d}(t) = \tilde{d}_0(t) - \frac{1}{\Theta}\tilde{x}(t)$$

(11)

In this approximation the unit sales are (nearly) equal to the generation rate of demanded units diminished by the rate $\tilde{x}(t)/\Theta$. Applying Eq.(1) and Eq.(6) in Eq.(11) we get for the total number of demanded units:

$$\tilde{x}(t) = \frac{\tilde{x}_0(t)}{1 + \Theta\eta\tilde{z}(t)}$$

(12)

Expanding this relation for small $\tilde{z}(t)$ yields:

$$\tilde{x}(t) \cong \tilde{x}_0(t)\left(1 - \Theta\eta\tilde{z}(t)\right)$$

(13)

[4] The impact of the finite lifetime of durable goods is neglected because it is large compared to non-durables (see [8]).

[5] This approximation is known as adiabatic approximation [14].

It suggests that for small $\tilde{z}(t)$ there is a large number of demanded units $\tilde{x}(t) \approx \tilde{x}_0(t)$ but decreases with increasing $\tilde{z}(t)$. In order to determine the time evolution of $\tilde{z}(t)$ we apply Eq.(13) in Eq.(1). Then Eq.(8) becomes:

$$\frac{d\tilde{z}(t)}{dt} = \alpha(t)\tilde{z}(t) - \Theta\eta\alpha(t)\tilde{z}(t)^2$$

(14)

with:

$$\alpha(t) = \eta\gamma\tilde{x}_0(t)$$

(15)

Approximating the function $\alpha(t)$ by the time average over the considered time interval Δt:

$$\alpha = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t}\alpha(t)dt$$

(16)

Eq.(14) becomes a logistic differential equation with constant coefficients. The evolution of the number of supplied units can be given by:

$$\tilde{z}(t) = \frac{z_{\max}}{1 + C_z e^{-\alpha t}}$$

(17)

with the integration constant C_z and:

$$z_{\max} = \frac{1}{\eta\Theta}$$

(18)

For a supply market with $\gamma \sim \alpha > 0$, Eq.(17) predicts that the total number of available units $\tilde{z}(t)$ increases in time according to a logistic law until $z(t)=z_{max}$. When $\tilde{z}(t)$ approaches the maximum magnitude z_{max}, the number of demanded units is defined by $\tilde{x}(t) = \tilde{x}_0(t)/2$. This can be seen by inserting $z(t)=z_{max}$ in Eq.(12).

The total unit sales Eq.(11) have with Eq.(12) and Eq.(17) the form:

$$\tilde{y}(t) = \frac{\tilde{d}_0(t)z(t)}{z_{\max}} = \frac{\tilde{d}_0(t)}{1 + C_z e^{-\alpha t}}$$

(19)

This relation suggests that the sales evolution is determined by the generation rate of demanded units $\tilde{d}_0(t)$ and the evolution of available units $\tilde{z}(t)$. At introduction of the good $\tilde{z}(t)$ is necessarily small and therefore the unit sales are limited by the amount of available units. It implies that there is a large contribution of unsatisfied demands in this period of the

PLC, referred to as lost sales.[6] When the total supply increases until $\tilde{z}(t)=z_{max}$ the total unit sales are equal to the demand rate:

$$\tilde{y}(t) \cong \tilde{d}_0(t)$$
(20)

and lost sales disappear. However, in this state suppliers cannot sell more units per unit time than generated by the demand rate $\tilde{d}_0(t)$, even if they would increase $\tilde{z}(t)$ above z_{max}. For constant z_{max} Eq.(3) suggests that then:

$$\tilde{d}_0(t) = \tilde{s}(t)$$
(21)

It means, when $\tilde{z}(t)$ reaches its stationary state the supply flow is equal to the demand rate, which expresses market equilibrium in the neo-classic theory. However, the demand rate $\tilde{d}_0(t)$ is not explicitly known.[7] The only empirically available variable is $\tilde{y}(t)$. However, as will be shown in the next chapter, the evolution of $\tilde{z}(t)$ has an impact on the mean price of the good.

2.1.2. The Price Evolution of Homogeneous Goods

In order to take the price p as decision variable of potential buyers into account we treat the aggregated number of demanded and supplied units $x(p,t)$ and $z(p,t)$ as price dependent functions. Generalizing Eq.(1) we assume that the number of sold units in a given price interval must disappear if the corresponding numbers of $x(t,p)$ or $z(t,p)$ vanish. Hence the price dependent unit sales are up to a first order approximation proportional to both variables:

$$y(t,p) \cong \eta x(t,p) z(t,p)$$
(22)

where the meeting rate η is treated as price independent. The price dispersion of sold units of a homogeneous good is determined by the probability density $P_y(t,p)$ given by:

$$P_y(t,p) = \frac{y(t,p)}{\tilde{y}(t)}$$
(23)

As established in [13] the price dispersion of homogeneous goods can be approximated for short time horizons by a symmetric Laplace distribution of the form:

$$P_y(p) \cong \frac{1}{2\sigma} e^{-\frac{|p-\mu|}{\sigma}}$$
(24)

[6] This effect is also known as backordering.

[7] An indication of demanded units $\tilde{x}(t)$ can be given by the size of waiting lists for the good.

with the standard deviation:

$$Std\left(P_y(p)\right)=\sqrt{2}\sigma \cong \sqrt{2}\left(\mu-\mu_m\right)$$

(25)

where $\mu_m>0$ is a minimum mean price indicating a technological limit beyond which the supply of product units is not profitable. Further derived is a relation that governs the mean price dynamics of this price dispersion. It is shown that the mean price can be determined by a Walrus equation of the form:

$$\frac{1}{\mu-\mu_m}\frac{d\mu}{dt}=H\left(\frac{d\tilde{x}}{dt}-\frac{d\tilde{z}}{dt}\right)$$

(26)

while $H>0$ is treated as a constant. This relation can be used to characterize the evolution of the mean price of nearly homogeneous durable goods. For this purpose we take advantage from Eq.(10) and approximate:

$$\frac{1}{\mu(t)-\mu_m}\frac{d\mu(t)}{dt}\cong -H\frac{d\tilde{z}(t)}{dt}$$

(27)

That means, in a polypoly fast growing market the mean price is essentially determined by the evolution of available product units. Applying Eq.(14) we further get:

$$\frac{d\mu(t)}{dt}\cong -H\alpha\tilde{z}(t)\left(\mu(t)-\mu_m\right)$$

(28)

while higher order terms in $\tilde{z}(t)$ are neglected. Since $\alpha>0$ the mean price of the durable market declines as a result of the excess supply. The stationary solution of this relation is given either by $\mu=\mu_m$ or $\tilde{z}=z_{\max}$. Eq.(25) suggests that for $\mu=\mu_m$ the standard deviation disappears and market becomes a monopoly market. Since we focus here on polypoly markets, this case is not further considered.

For $\mu(t)>\mu_m$, Eq.(27) can be written as:

$$\int\frac{d\mu(t)}{\mu(t)-\mu_m}\cong -H\int d\tilde{z}(t)$$

(29)

and we readily obtain:

$$\mu(t)=\mu_0 e^{-H\tilde{z}(t)}+\mu_m\cong \mu_0\left(1-H\tilde{z}(t)\right)+\mu_m$$

(30)

The model suggests therefore that for the considered market constellation the mean price declines with increasing $\tilde{z}(t)$ given by Eq.(17). For $\tilde{z}(t)\rightarrow z_{\max}$ the mean price approaches a floor price $\mu_f>\mu_m$ according to a logistic law determined by:

$$\mu_f = \mu_0 \exp(-Hz_{\max}) + \mu_m$$

(31)

The introduction mean price of the good $\mu(0)$ is defined by:

$$\mu(0) = \mu_0 \exp(-H\tilde{z}(0)) + \mu_m$$

(32)

while generally $\tilde{z}(0) \neq 0$.

2.2. The Product Lifecycle of Homogeneous Durable Goods

The unit sales evolution of durable goods is usually referred to as product lifecycle. The unit sales are essentially determined by first- and repurchase of the durable. First purchase is related to the spreading of the good into the market known as market diffusion. This spreading process can be described by the cumulative number of adopters $N_A(t)$. In order to be in line with previous research we want to define the market penetration $n(t)$ by[8]:

$$n(t) = \frac{N_A(t)}{M}$$

(33)

The number of all possible adopters in a given domain interested in purchasing the consumer durable is termed market potential M.[9]

We want to take the inhomogeneity of the demand side of the market with respect to the price into account by introducing a market volume $V(\mu) \leq M$. It represents the number potential adopters who can afford the good for a mean price μ. The scaled market volume becomes:

$$v(\mu) = \frac{V(\mu)}{M}$$

(34)

while $0 \leq v(\mu) \leq 1$.

First purchase is determined by the time evolution of the number of adopters. However, the number of adopters is governed by a conservation relation of the form:

$$\frac{dn(t)}{dt} = \varphi(t)\psi(t) - \theta n(t)$$

(35)

The first term indicates the generation of adopters. It is proportional to the generation rate $\varphi(t)$ and the number of potential adopter $\psi(t)$ not yet adopted the good. This number can be written as the difference between the market volume $v(\mu)$ and $n(t)$:

$$\psi(t) = v(\mu) - n(t)$$

[8] As mentioned above integer numbers are scaled by a large number. In order to establish a consistent model this large number is M.

[9] For simplicity M is treated as time independent.

(36)

The second term in Eq.(35) indicates the decline of $n(t)$ with a mean decline rate θ. In the initial stages of the lifecycle the decline rate is $\varphi(t)>0$ while $\theta(t)\approx 0$. In the decline phase is $\varphi(t)=0$, $\theta\neq 0$. Expanding the generation rate $\varphi(t)$ as a function of the number of adopters leads to:

$$\varphi(t) \cong \mathrm{A} + \mathrm{B}n(t)$$

(37)

with constant coefficients $A, B>0$. Inserting Eq.(37) in Eq.(35) with $\theta=0$, we obtain a standard model describing the diffusion process of goods known as the Bass model [16]. The first purchase unit sales are determined by the growth of the market penetration:

$$\tilde{y}_f(t) = \frac{dn(t)}{dt} = \mathrm{A}\big(v(\mu) - n(t)\big) + \mathrm{B}n(t)\big(v(\mu) - n(t)\big)$$

(38)

The first term is interpreted as spontaneous purchase by potential adopters with the so-called innovation rate A. The second term is due to social learning, where the number of adopters $n(t)$ increases with an imitation rate B.

Repurchase events must be proportional to the current number of adopters. The total repurchase $\tilde{y}_r(t)$ can therefore be modelled as the product of $n(t)$ with a time dependent repurchase rate $\xi(t)$ characterizing the average number of repurchased units per unit time and adopter:

$$\tilde{y}_r(t) = \xi(t)n(t)$$

(39)

The repurchase rate $\xi(t)$ may increase in the run of time with the economic evolution of the considered country. Since $\xi(t)$ cannot grow up to infinity we demand that it approaches a maximum magnitude ξ_{max} after sufficient time. As a first order approximation such a constraint growth can be described by a logistic differential equation of the form:

$$\frac{d\xi(t)}{dt} = a\xi(t)\left(1 - \frac{\xi(t)}{\xi_{\max}(t)}\right)$$

(40)

where $a>0$ characterizes the growth of the repurchase rate. The solution of this relation determines repurchase rate:

$$\xi(t) = \frac{\xi_{\max}}{1 + C_\xi e^{-at}}$$

(41)

with the free parameter C_ξ.

The total unit sales are in the approximation Eq.(11) equal to the sum of first- and repurchases:

$$\tilde{y}(t) \cong \tilde{d}(t) = \tilde{y}_f(t) + \tilde{y}_r(t) = \frac{dn(t)}{dt} + \xi(t)n(t)$$

(42)

The product lifecycle can be characterized by the main process governing the adopter evolution:

i) In the introduction phase of the lifecycle the main source of the increase of *n(t)* is spontaneous purchase. Hence, in the introduction phase the first term in Eq.(38) dominates over the second which leads to the condition $A>Bn(t)$.

ii) In the growth phase of the PLC social learning dominates the adopter evolution. Thihs phase is therefore characterized by $A<Bn(t)$. At the transition between both phases the number of adopters is $n_g \approx A/B$. The penetration n_g can be interpreted as the minimum number necessary to start the growth phase of the PLC.

iii) In the maturity phase the spreading due to social contagion saturates while:

$$\frac{dn(t)}{dt} \approx 0$$

(43)

From Eq.(38) flows with Eq.(43) that the number of adopters is in this period:

$$n(t) \cong v(\mu(t))$$

(44)

That means, when the fast spreading process due to social contagion slows down the number of adopters increases with the market volume $v(\mu(t))$. Since $\mu(t)$ is not constant an additional diffusion process occurs due to the decline of the mean price. However in order to keep the model simple this additional price dependent diffusion process in the maturity phase is not further taken into account here. [10] Instead we approximate the market volume by a constant $v(\mu) \approx n_0$. The evolution of the market penetration *n(t)* can then be described by the solution of Eq.(38) given by:

$$n(t) = \frac{1 - e^{-(A+B)t}}{\left(1 + \frac{B}{A} e^{-(A+B)t}\right)^2} n_0$$

(45)

which yields an *S*-curve in time. The total first purchase demand caused by Bass diffusion becomes:

$$\tilde{y}_f(t) = \frac{dn(t)}{dt} = \frac{A(A+B)^2 e^{-(A+B)t}}{\left(A + Be^{-(A+B)t}\right)^2} n_0$$

(46)

[10] Models including the price dependent market volume are discussed for example in [10,15]. It is shown that for an exponential decrease of the mean price as can be found in this model the diffusion process can be described by Gompertz equation.

iv) The decline phase is related to the occurrence of a close substitute of the durable at time step t_d. While in the initial stages of the lifecycle the parameter a in Eq.(40) is positive it changes after the introduction of the substitute. Since potential buyers are no longer interested in repurchasing the current good we assume that the positive amplitude a becomes at t_d a negative amplitude $-a'$, while $a'>0$. Eq.(40) turns therefore for $t>t_d$ into:

$$\frac{d\xi(t)}{dt} \cong -a'\xi(t) \quad (47)$$

neglecting higher terms in $\xi(t)$. It has the solution:

$$\xi(t) = \xi(t_d)e^{-a'(t-t_d)} + \xi_{\min} \quad (48)$$

where ξ_{min} indicates a remaining minimum repurchase rate. As a result of the declining repurchase rate the number of adopters declines also with the rate $\theta>0$ but slower than the unit sales. Applying Eq.(35) with $\varphi(t>t_d)=0$ we get for $t>t_d$:

$$n(t) = n(t_d)e^{-\theta(t-t_d)} + n_{\min} \quad (49)$$

while n_{min} is an eventually remaining minimum number of adopters.

With these relations the entire product lifecycle can be compared with empirical data with a minimum set of free parameters. The evolution of the number of available units $\tilde{z}(t)$ requires three free parameters z_{max}, α and C_z. For the mean price evolution three additional parameters are necessary μ_0, μ_f and H (For simplicity it is set $\mu_m=0$). The adopter evolution is characterized here by Bass diffusion Eq.(45). It involves the free parameters A, B and n_0. The decline phase starts at t_d and needs two additional parameters θ and n_{min}. The total unit sales $\tilde{y}(t)$ are governed in this model by Eq.(42) determined by the parameters of Bass diffusion and the repurchase rate parameters a, ξ_{max} and C_ξ. The decline phase finally uses the parameters a' and ξ_{min}. Since the empirical unit sales are usually given in absolute numbers, for a comparison the market potential M has to be known. A complete description of the product lifecycle of a durable requires in the presented approximation a minimum number of 19 free parameters.

2.3. The Relation between Supply and Diffusion

Eq.(19) suggests that the initial stages of the lifecycle suffer from a constrained supply. This effect may have an impact on the diffusion velocity of a durable. In order to estimate the impact of a constrained supply on the imitation parameter, we approximate the total unit sales by the first purchase sales in the growth phase of the lifecycle neglecting the introduction phase. They must be equal to Eq.(19):

$$\tilde{y}(t) \cong \tilde{y}_f(t) \cong \mathrm{B}n(t)\psi(t) = \frac{\tilde{d}_0(t)}{1+C_z e^{-\alpha t}} \quad (50)$$

Obviously the impact of the constraint supply can be neglected for $C_z=0$. It corresponds to the case that the total amount of available units is always z_{max}. The idea is to treat B as a function of C_z and expand the imitation parameter $B(C_z)$ around zero up to the first order as:

$$\mathrm{B}(C_z) \cong B_0 + B_1 C_z \quad (51)$$

with the unknown coefficients B_0 and B_1. Inserting this relation in Eq.(50) and expanding the also the right hand side of Eq.(19) for small C_z we get:

$$(B_0 + B_1 C_z) n(t)\psi(t) \cong \tilde{d}_0(t)\left(1 - C_z e^{-\alpha t}\right) \quad (52)$$

A comparison of the coefficients of the corresponding order in C_z leads to:

$$B_0 n(t)\psi(t) = \tilde{d}_0(t); \qquad B_1\, n(t)\psi(t) = -\tilde{d}_0(t) e^{-\alpha t} \quad (53)$$

It suggests that B_1 dependents on time and has the form:

$$B_1 = -B_0 e^{-\alpha t} \quad (54)$$

Taking the time average over the growth phase of the lifecycle Δt_g, we obtain for the mean magnitude of the imitation parameter $<B>$:

$$\langle B \rangle = \frac{B_0}{\Delta t_g} \int_0^{\Delta t_g} \left(1 - C_z e^{-\alpha t}\right) dt = B_0 (1 + \beta) \quad (55)$$

where:

$$\beta = \frac{C_z}{\alpha \Delta t_g}\left(e^{-\alpha \Delta t_g} - 1\right) \quad (56)$$

The model suggests in this approximation a linear relationship between the imitation parameter and β. The supply constraint has no impact on the imitation parameter if $\beta=0$. In this case the imitation parameter has its maximum magnitude B_0. Note that β is negative since the exponential function is smaller than one for $\Delta t_g>0$. Eq.(55) suggest that the magnitude of B decreases with increasing C_z. This is because the initial magnitude of available units decreases with increasing C_z. The imitation parameter becomes also small if the rate α is small in the period Δt_g. In other words, if the output of the suppliers rises very slowly over the considered growth period, the diffusion process is delayed.

Note that the presented consideration of a delayed market diffusion is not a consequence of the price evolution. That the price declines for $\alpha>0$ is rather a side effect. The main cause of a delayed diffusion is the constrained supply in the initial stages of the lifecycle accompanied with lost sales.

3. Comparison with Empirical Results

The presented model suggests that the PLC of nearly homogeneous consumer durables in polypoly markets have the following characteristics:
1. Homogeneous durable goods in polypoly markets have a price dispersion that can be approximated by the Laplace distribution Eq.(24).
2. For a fast growing supply the number of available units $\tilde{z}(t)$ increases in time according to the logistic law Eq.(17). This evolution causes a decline of the mean price in time described by Eq.(30).
3. Applying the Bass model for the initial stages of the lifecycle the market penetration $n(t)$ can be approximated by Eq.(45) and by Eq.(49) for the decline phase.
4. The total unit sales are determined on the one hand by the evolution of the number of adopters and on the other hand by the repurchase of the good. The unit sales are given by Eq.(46) for first purchase and by Eq.(39) for repurchase with the repurchase rate Eq.(41). The decline phase is governed by an exponential decay of the repurchase rate with the repurchase rate Eq.(48).
5. The diffusion processes is subject to a supply constraint. This constrained leads on time average to a decrease of the maximum imitation rate B_0. Plotting the imitation rate B as a function of β suggests a linear relationship.

In order to compare the presented model with available empirical data a number of consumer durables are studied satisfying the model conditions.[11] For the chosen samples both the market penetration and the price evolution is known. For two samples are also data for the unit sales available. However, the first assertion cannot be verified for the considered samples because the price dispersion is of these goods are not available. Though, from empirical data of the price dispersion of homogeneous goods it is known that their price dispersion can be approximated by a Laplace distribution [13,17]. It is assumed that the chosen examples can be treated as nearly homogeneous.

3.1. Market penetration, Mean Price and Supply Evolution

In order to illustrate the entire PLC of a durable, the empirical market penetration $n(t)$ of Black & White (B&W) TV sets in the UK [18] is displayed in Fig.1 (squares) together with a fit of the Bass model Eq.(45) for the initial stages of the PLC and with a fit of Eq.(49) for the decline phase with the parameters summarized in Table 1 (solid line). Indicated are the phases characterizing the stages of the lifecycle as suggested by the model. The transition between introduction and growth phase takes place for small market penetrations $n_g \approx 2\%$. Neglected in this model is the increase of the market volume with decreasing price in the maturity phase. We assume that deviations from the Bass model Eq.(45) in the slow down period indicates its start of this penetration process. Data points not matching Eq.(45) are not further discussed here. As expected, the decline phase of this good can be described by the exponential function Eq.(49) starting at $t_d=26$ years.

Also displayed is an average sales price $\mu(t)$ scaled by the maximum price [19]. The dashed line represents a fit of Eq.(30) with the parameters in Table 1. Displayed in the insert

[11] Not considered are for example obvious heterogeneous goods like the personal computer. It comprises of a competing technological generations of the same good.

is the function $\tilde{z}(t)$ with $z_{max}=1$ that follows from this fit indicating the amount of freely available units. Another example of Black & White TV sets (USA) is shown in Fig.2 [19]. Both have a similar dynamics, but the data of the decline phase of US-Market are not available.

Presented in Fig.3 is the market penetration and price evolution of CD-players in the USA [20]. This good has the smallest parameter $\alpha=0.17$ in the considered collection of durables. It suggests a slow capacity increase in time. A consequence of the presented theory is that for small α the mean price is susceptible for demand and supply fluctuations. This can be seen in the initial phase of the market penetration where a considerable fluctuation of the mean price is evident. Approaching the inflection point of the adopter evolution (maximum of the first purchase unit sales) price fluctuations settle down. A very fast growing market is that of DVD players in the USA displayed in Fig.4 [20]. This market has the highest growth rate $\alpha=0.65$ exhibiting a rapid increase of supplied units accompanied with a considerable price decline.

Displayed in Fig.5 is the market evolution of Color TV sets in the USA [21]. Note that the Bass model applies merely up to about *80%* of the total market potential. The presented model is based on the idea of a constant α over the entire lifecycle Δt (Eq.(16)). The empirical data suggest, however, that the supply evolution may have separated periods with nearly constant α, such that the price evolution can be approximated by two logistic waves. This can be interpreted as a rapid increase of production capacities after introduction followed by a much slower expansion. This seems to be also the case for the following examples. Applying two successive logistic waves the price and hence the supply evolution can be fitted with a higher accuracy. The data of the fit parameters from the price evolution are summarized in Table 1.

A similar market situation occurs for cell phones in the USA shown in Fig.6 [20]. The data suggest a considerable increase of the output up to 1991 accompanied with accompanied by rapid price decline. After this growth period the expansion of the capacities slowed down. A dichotomy of the supply evolution can be also found for the market evolution of VCR's in the USA [20]. Here the supply evolution consists of two growth waves of nearly equal α.

3.2. The Unit Sales Evolution

Beyond market penetration and price evolution the model also allows a comparison of the evolution of the total unit sales with available empirical data. For two examples data are available to perform the comparison. Displayed in Fig.8 are the unit sales of VCR's in the USA (squares) together with a fit of Eq.(46) and Eq.(39) for first and repurchase (solid line) with the parameters given in Table 1 [20]. The model suggests that the sales peak with its maximum around 1986 is due to first purchase caused by Bass diffusion. With the increase of the number of adopters and the growth of the repurchase rate the unit sales growth until 2000 to a maximum magnitude. The decline of the unit sales starts in 2001 caused by the introduction of DVD recorders as a close substitute.

Another example of the PLC of a durable good is displayed in Fig.9. It shows the rise and decline of the unit sales of CD-players in the USA (squares) and fit with the model equations (solid line) applying the parameters in Table 1 [20]. In difference to the VCR case the first sales peak due to the diffusion process in not clearly evident. This is due to the rather slow penetration of this good. The time until it is replaced by a close substitute (MP3 player) is much shorter than for VCR's. The decline rate a' of CD-Players is, however, much smaller than for VCR's suggesting that CD-Players can be found in US households over long time horizons.

3.3. The Relation between Product Lifecycle and Supply

Eq.(55) predicts a linear relationship between the imitation parameter B as a measure of the market diffusion velocity and the parameter β containing the impact of lost sales from a constrained supply flow in the initial stages of the lifecycle. Taking advantage from the data in Table1 the relation between both parameters is plotted in Fig.10.[12] From a linear regression fit we get for the solid line displayed in Fig.10 $B_0=1.7$ and $B_1=1.1$. Eq.(55) suggest, however, that both parameters should be equal. Since this is not exactly the case and due to the small coefficient of determination ($R^2=0.8$) of the regression fit we can conclude that the established theoretical considerations leading to Eq.(55) are qualitatively correct, but cannot verified quantitatively with this small number of examples satisfying the model conditions.

4. Discussion

Merging the lifecycle concept with the supply evolution some key relationships characterizing the PLC of durable homogenous goods in polypoly markets can be established. They are schematically displayed in Fig.11. At introduction the total number of available units $\tilde{z}(t)$ of the durable good is small, while the total number of demanded units $\tilde{x}(t)$ is high as indicated by the solid and dashed lines in this figure. It implies a high introduction mean price $\mu(0)$. In this state unsatisfied demands lead to a high number of lost sales. The aggregated demand and supply curves $x(p,t)$ and $z(p,t)$ at this time step are indicated by solid lines in the inserts of Fig.11. The model is based on the idea that the unit sales $y(p,t)$ are the result of the meeting of demanded and supplied units in a given price interval. Hence, $y(p,t)$ and also the price dispersion $P_y(p)$ have a maximum at mean price $\mu(t)$ where the functions $x(p,t)$ and $z(p,t)$ have maximum overlap. The resulting price dispersion $P_y(p)$ has for a homogeneous good always the form of a Laplace distribution (dotted line). In other words, the majority of product units of homogeneous goods are sold for mean price.

For a fast growing supply (supply market) the functions $x(p,t)$ and $z(p,t)$ are subject to time-dependent variations causing a shift of the mean governed by a Walrus equation. The fast increase of the total number of available units $\tilde{z}(t)$ leads to a slow decrease of $\tilde{x}(t)$ and a decline of mean price $\mu(t)$ (For some durables this evolution takes place in two waves separating at t_1). The function $\tilde{z}(t)$ growths governed by a logistic law until z_{max} is reached accompanied with a logistic decline of the mean price approaching the floor price μ_f. The price dispersion does not only shift to lower values in this process, since the price is strictly positive $P_y(p)$ becomes also a narrow peak approaching μ_f[13].

Accompanied with this evolution of supply and demand the good spreads into the market described here by Bass diffusion. This spreading process determines the evolution of the market penetration $n(t)$ and total unit sales $\tilde{y}(t)$ governed by the characteristic stages of the lifecycle indicated in Fig.11. In order to complete the description of the PLC the Bass model is supplemented by a logistic growth of the repurchase sales.[13] The decline phase starts with the introduction of a close substitute at t_d and is modelled as a negative repurchase rate causing an exponential decline of the total unit sales and (occasionally with a short time shift) also of the number of adopters.

Further discussed is the impact of the supply constrained at introduction of the lifecycle causing lost sales. The model predicts that the diffusion process is slowed down by a supply constraint, if the initial number of available units is small (high C_z) and when the growth of supplied units during the growth period Δt_g is small (small α). The comparison with empirical investigations show that the theoretical considerations leading to a linear

[12] In the case of two supply waves, the data of the first supply wave with index 0 are taken.
[13] Periodic oscillations in the repurchase process due to the initial growth period are neglected (see [15]).

relationship between the imitation parameter *B* and a comprised parameter β indicating the supply growth are qualitatively correct.

It has to be emphasized that the effect of a delayed diffusion is caused by the supply flow and not by price changes as suggested by previous considerations [22]. The point is that there is always a distribution of the willingness to pay described in this model by the aggregated supply function $x(p,t)$. But adopters contributing to Bass diffusion (market volume) are treated in this model as price independent. Therefore the spreading process is in the first place a price independent process having its origin in social contagion. The decrease of the mean price is rather a side-effect. The main cause of a delayed diffusion is according to this theory the occurrence of lost sales in the initial stages of the PLC.

5. Conclusions

We can conclude:

1. The PLC of durable goods can be understood as governed by two processes: the spreading of the good into market and the relation between supply and demand determining the mean price. The presented dynamic model allows a description of the entire lifecycle of homogeneous durables in polypoly markets.
2. Both processes interfere in the initial stages of the PLC. The spreading of the good into the market is delayed due to lost sales. The delay increases with a small number of available units at introduction and a slow output increase.
3. The presented considerations suggest that the forecast of the PLC of durable has to take the evolution of supply side into account.

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Tables

Parameter	***B&W TV (UK)***	***B&W TV (USA)***	***CD Player***	***DVD Player***	***Colour TV***	***Cell Phone***	***VCR***
Figure	1	2	3	4	5	6	7
t_0	1946	1948	1983	1998	1954	1984	1978
α [$year^{-1}$]	0.3	0.55	0.17	0.65	0.38	0.6	0.6
z_{max}	1	1	1	1			
C_z	8	10	6	4			
z_{max0}					0.28	0.5	0.5
C_{z0}					10	10	10
α_0 [$year^{-1}$]					0.38	0.6	0.6
t_1 [years]					19	5	10
z_{max1}[14]					0,72	0.5	0.5
C_{z1}					6	10	150
α_1 [$year^{-1}$]					0.2	0.43	0.63
μ_0 [%]	130	110	118	150	100	112	105
μ_f [%]	60	28.3	15	20	8	14	8
H	0.77	1.35	2.06	2	2.52	2	2.57
A [$year^{-1}$]	0.0065	0.05	0.0085	0.007	0.0005	0.0015	0.0018
B [$year^{-1}$]	0.4	0.4	0.45	1.2	0.4	0.55	0.64
n_0 [%]	90	90	76	70	83	55	82
M in Mio			85				79
t_d[years]	26		17,8				23.8
θ[$year^{-1}$]	0.16						
n_{min}	0						
a[$year^{-1}$]			0.3				0.31
a'[$year^{-1}$]			0.1				0.65
ξ_{min}			0				0
ξ_{max}			1.0				0.33
C_{ξ}.			50				120
Δt_g[years]	20	14	18	10	25	15	16
β	-1.03	-1.29	-1.25	-0.57	-1.02	-1,1	-1.03

Table 1. Characteristic parameters of the studied examples.

Figures

[14] The time dependence in Eq.(17) has to be replaced for the second wave by t-t_1.

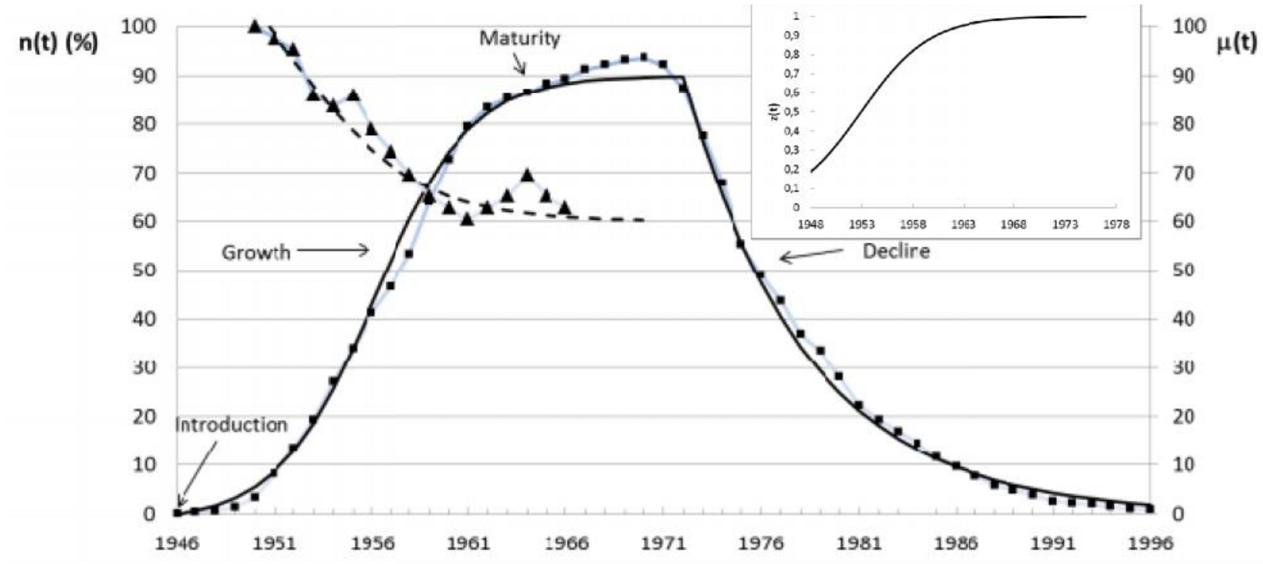

Figure 1: Evolution of the market penetration *n(t)* (squares) and mean price *μ(t)* (triangles) of Black & White TV sets in the UK [18,19]. The solid line (market penetration) and dashed line (mean price) are a fit with the parameters in Table 1. The insert shows the function $\tilde{z}(t)$ derived from the mean price evolution.

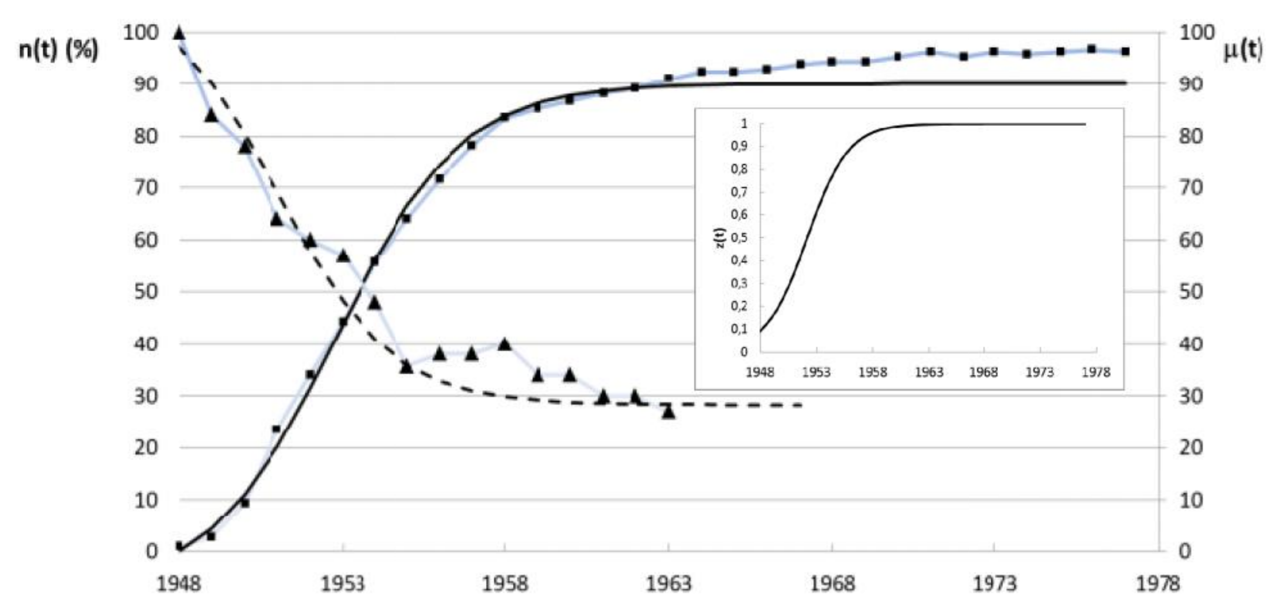

Figure 2: Evolution of the market penetration *n(t)* (squares) and mean price *μ(t)* (triangles) of Black & White TV sets in the USA [19].

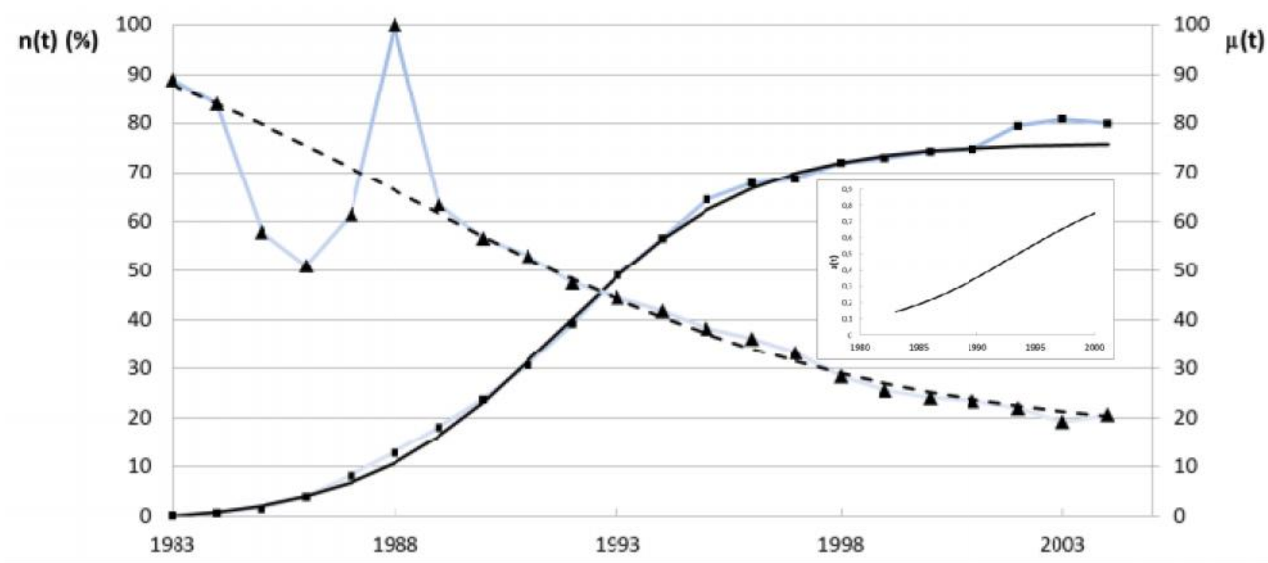

Figure 3: Evolution of the market penetration *n(t)* (squares) and mean price *μ(t)* (triangles) of CD players in the USA [20].

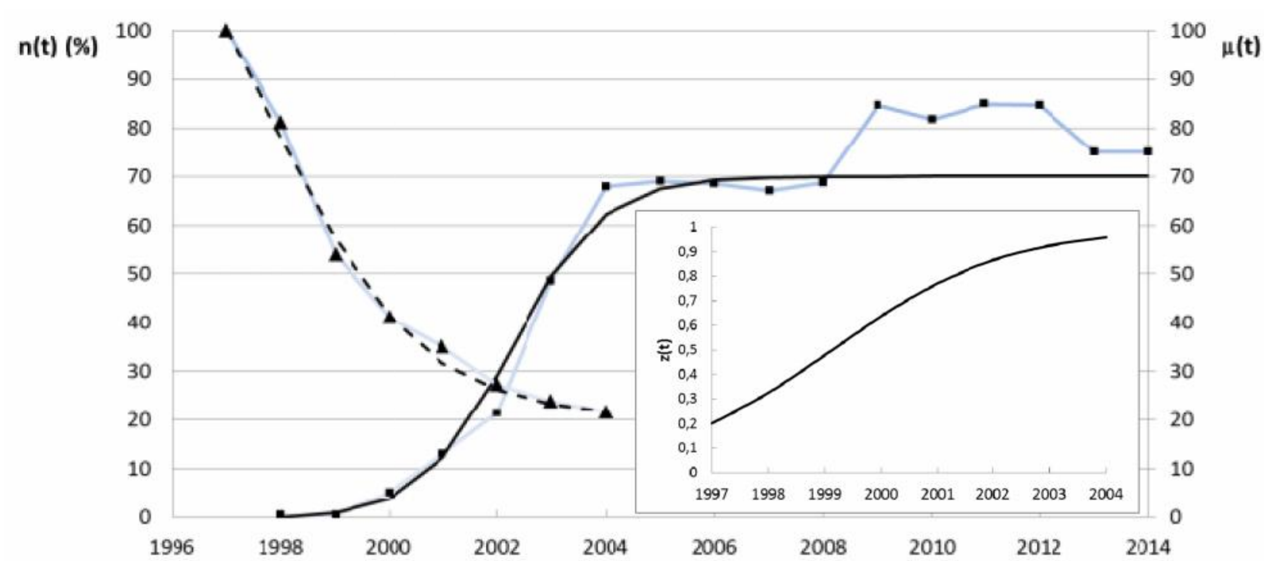

Figure 4: Evolution of the market penetration *n(t)* (squares) and mean price *μ(t)* (triangles) of DVD players in the USA [20].

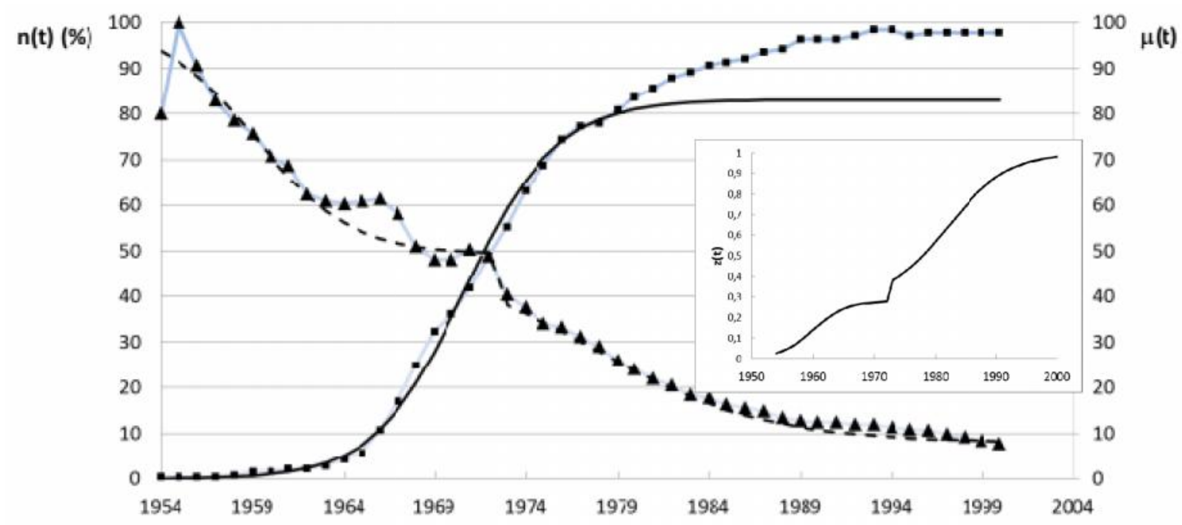

Figure 5: Evolution of the market penetration *n(t)* (squares) and mean price *μ(t)* (triangles) of Color TV sets in the USA [21].

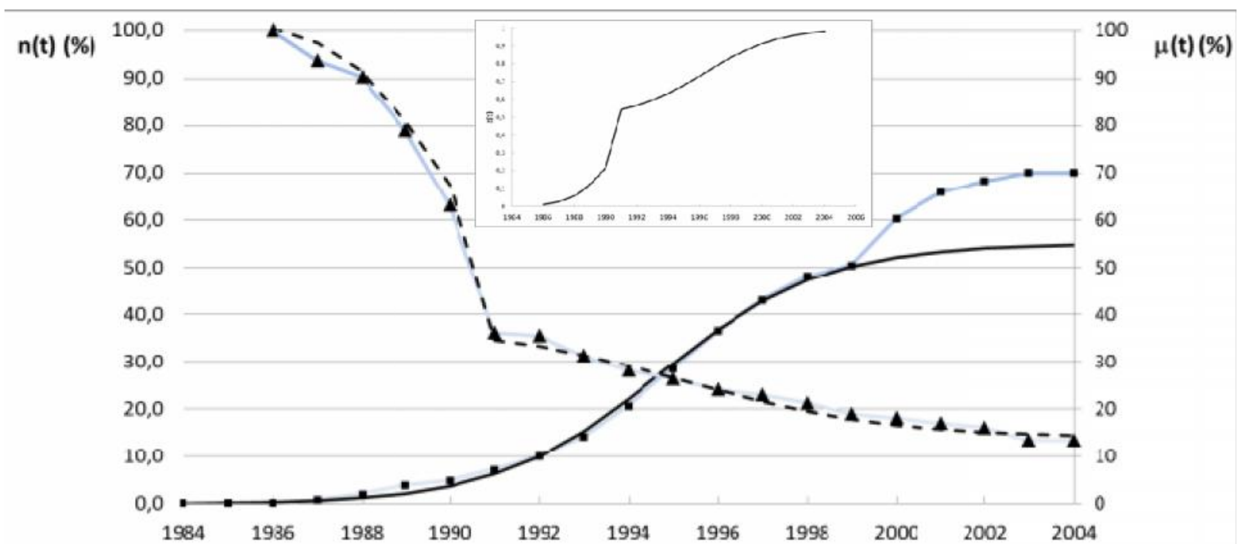

Figure 6: Evolution of the market penetration *n(t)* (squares) and mean price *μ(t)* (triangles) of cell phones in the USA [20].

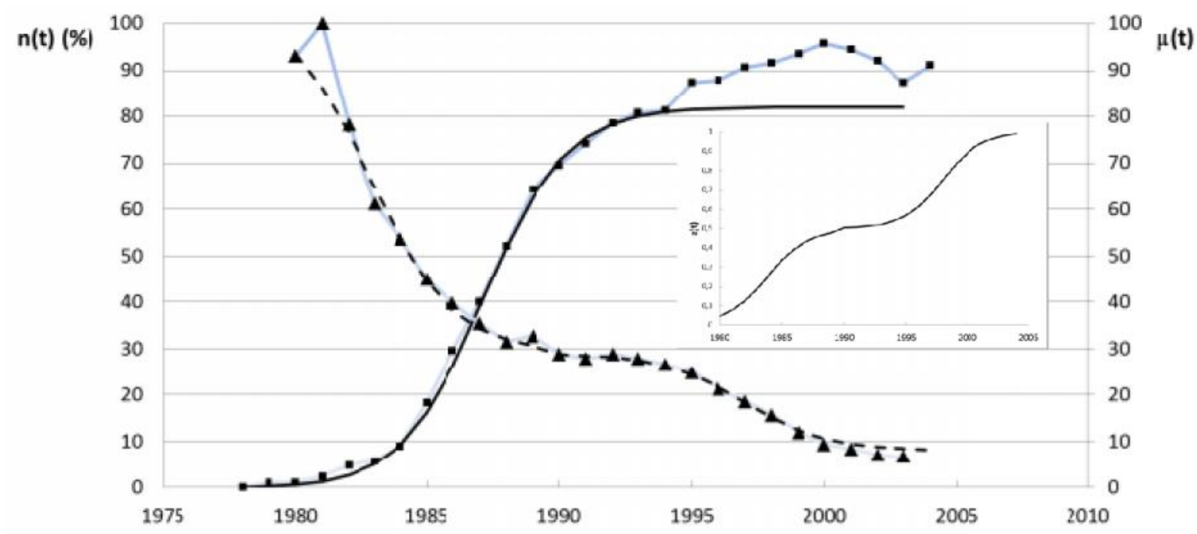

Figure 7: Evolution of the market penetration *n(t)* (squares) and mean price *µ(t)* (triangles) of VCR's in the USA [20].

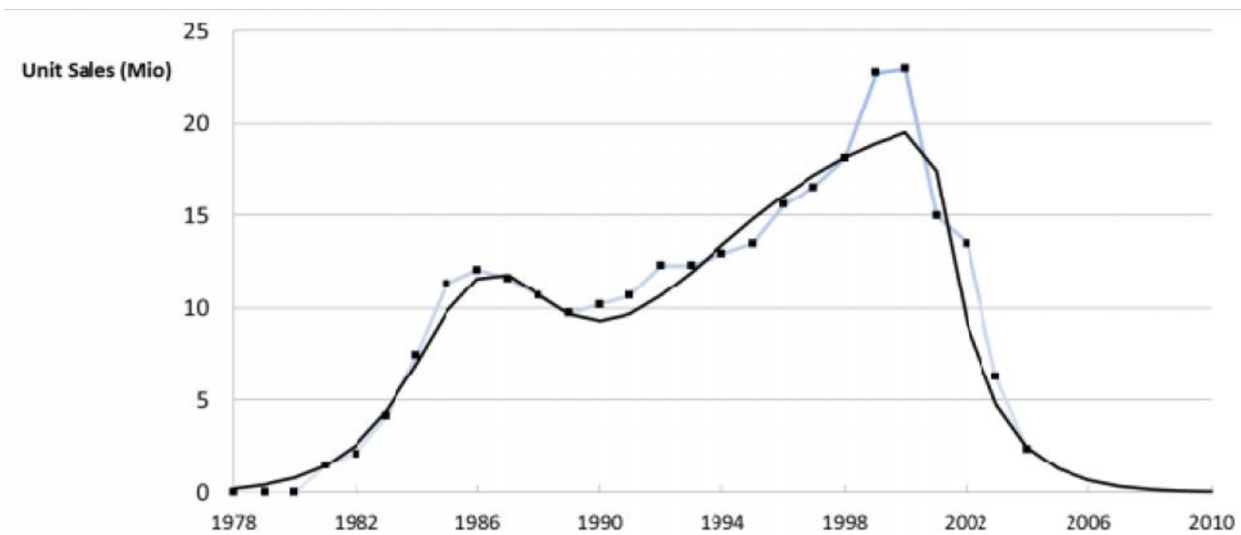

Figure 8: Evolution of the unit sales of VCR's (squares) in the USA [20].

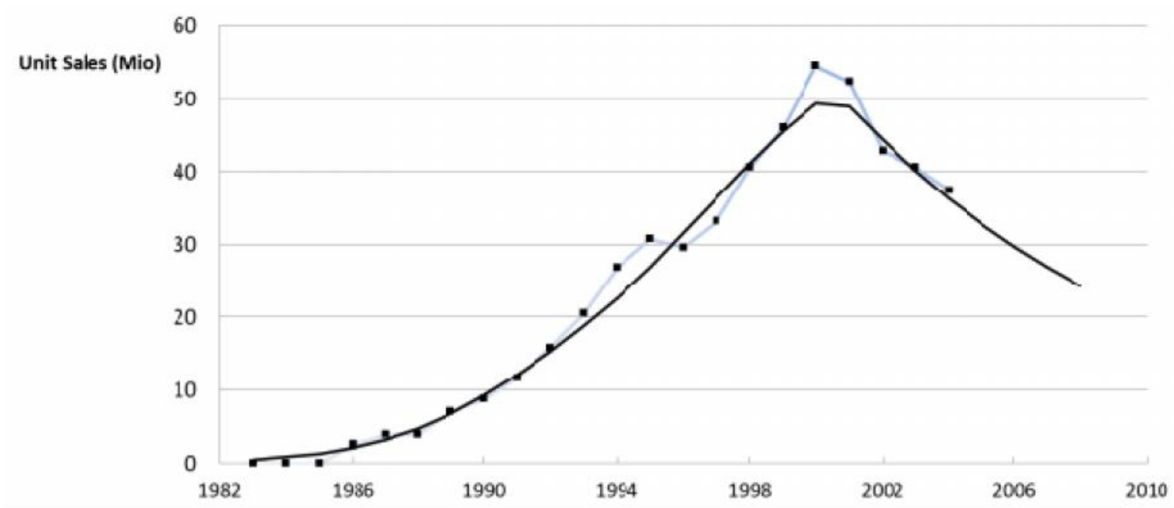

Figure 9: Evolution of the unit sales of CD players (squares) in the USA [20].

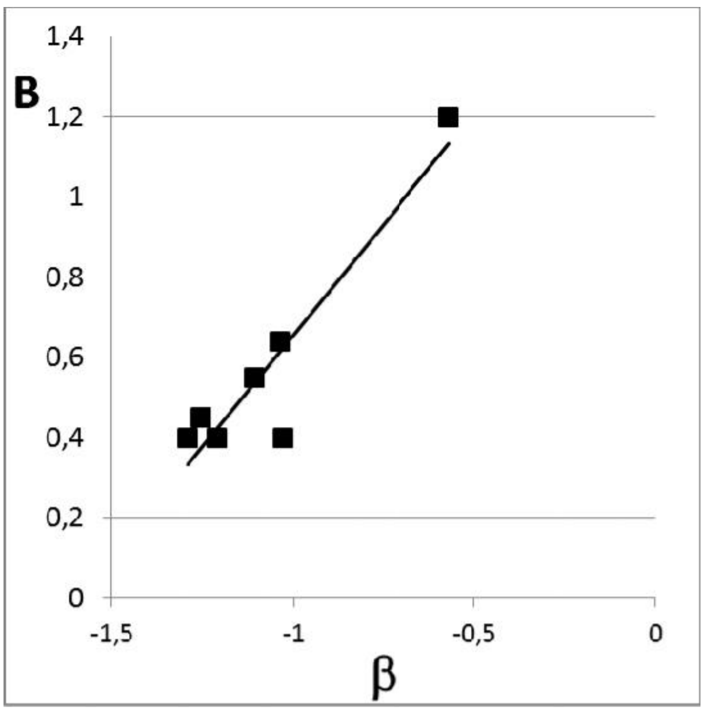

Figure 10: Displayed is the imitation parameter B (squares) as a function of β applying the data given in Table 1. The solid line is a linear regression function.

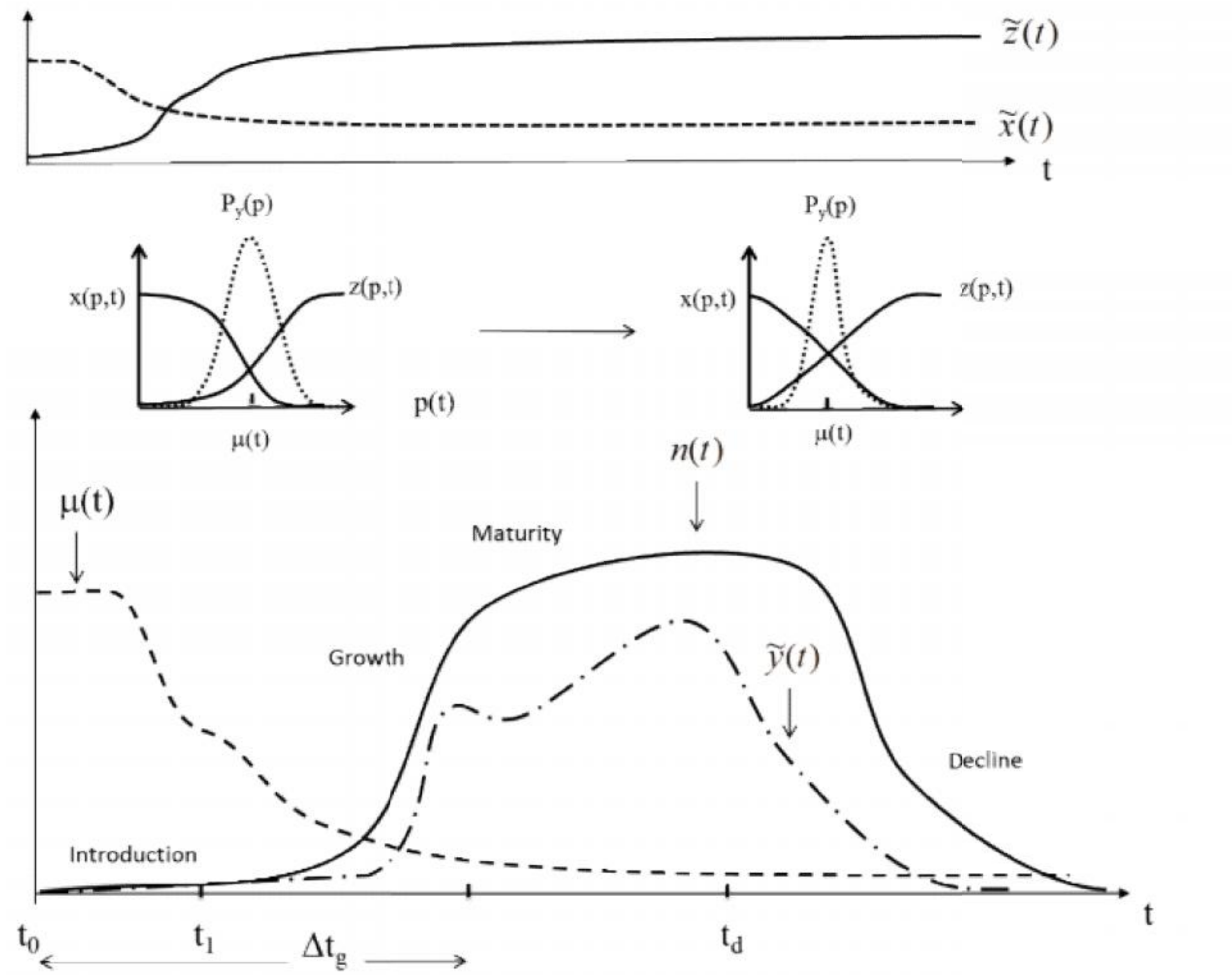

Figure 11: The product lifecycle of a durable homogenous good in polypoly markets suggested by the presented model.