MODELLING STUDENTS’ AFFINITY FOR LECTURE ATTENDANCE USING DIFFERENTIAL EQUATIONS: THE CASE OF STUDENTS OF KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

W. Obeng-Denteh, Odum Kodua-Nana, S. Owusu Ansah, R. Kwame Ansah, Alex Quashie Nyamadu

Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana

ABSTRACT

Research Context: The aim of the study was to investigate the students’ affinity for lecture attendance using differential equations and to explore reasons why students absent themselves from lectures.

Research Methods: The study employed a quantitative methodology which was adopted in order to allow the researchers to gather more precise and quantifiable information on students’ perception about lecture attendance. Questionnaires were administered continuously to a cross-section of students numbering five hundred to ascertain their affinity for lecture attendance and reasons why they attend or do not attend lectures.

Conclusion: A logistic differential equation had been used to model students’ affinity for lecture attendance. The calculated or predicted values of students who regularly attend lectures were obtained by using logistic differential equation. The predicted values were then compared with the observed values and analyzed. From tables 1, 2, 3 and the graphs in figures 1 and 2 which were obtained by the usage of Matlab, it was seen that the observed values and the predicted values almost converged to the same values, signifying that the model is good for the prediction of students’ preference for lecture attendance. It was interesting to note that the reasons that drive the majority of students to lectures are to find out what they are supposed to learn, not to miss important information and to find out about assessment task. The study also revealed that some students use on-line notes as a substitute for lecture attendance.

Keywords: differential equations, students, model, attendance, observed and calculated values
INTRODUCTION

A Differential equation [3] is an equation involving an unknown function and its derivatives. It can also be defined as equation containing the derivatives of one or more dependent variable(s) with respect to one or more independent variable(s).

The subject of differential equations is solving problems and making predictions [4]. In mathematics, the rate at which a quantity changes is the derivative of that quantity.

The tendency towards decreasing lecture attendance by students is a concern for many tertiary institutions. This is a phenomenon that is both interesting and frustrating and yet there is very little evidence of university or governmental policy on it. However where the policy exist, it certainly seems to differ not only from university to university but even from department to department. Despite the emphasis on quality and flexibility, and the introduction of new technologies, the lecture remains at the centre of most universities’ approaches to teaching and learning [1]. Poor lecture attendance may have a negative influence on students’ performance: as the semester progresses, poor lecture attendees may perform increasingly worse. When students are absent from lectures they miss valuable information resulting from peer-lecturer interaction and the benefits of the specific examples lecturers use to clarify difficult concepts. Absenteeism [5] on the part of students disturbs the dynamic teaching-learning environment and adversely affects the overall well-being of lectures.

In this problem, our main objective is to develop and analyse a differential equation model based on a given data on students’ affinity for lecture attendance and to demonstrate effective way of improving lecture attendance in the case of students of Kwame Nkrumah University of Science and Technology (KNUST).
METHODOLOGY

The study employed a quantitative methodology which was adopted in order to allow the researchers to gather more precise and quantifiable information on students’ perception about lecture attendance. The research design that was used in this study was the descriptive survey method. The descriptive survey is defined as “the method of research that simply looks with intense accuracy at the phenomena of the moment and then describes precisely what the research sees”. Descriptive survey is probably the best method which is available to use in collecting data for the purpose of describing a population large enough to observe directly. The main instrument that was used to solicit for information was the questionnaire method. Self-administered questionnaires with open-ended and closed questions were used for data collection.

The data was collected from both undergraduate and post-graduate students in KNUST from all the six (6) colleges during the second semester of the 2011/2012 academic year. In all, five hundred (500) students comprising 218 females and 282 males responded to the questionnaires. The questionnaires were administered continuously until the number was exhausted.

The logistic equation was used to model the data collected and matlab was also used in the data analysis.

LIMITATIONS OF THE STUDY

The main purpose of the study was to determine student affinity for lecture attendance. However, the fact that only those students who were surveyed were present at specific lectures, somewhat challenges the efforts to determine what reasons why students do not usually attend lectures may have provided to substantiate their view of lecture attendance. Some students were reluctant in providing us with the information needed.
THE LOGISTIC MODEL OF GROWTH

Exponential growth may be a good assumption provided it is not constrained either by space or by the availability of food (or some other resource). But of course all populations will be so constrained once they reach some size. A more ‘realistic’ model is given by introducing some limiting effects as the stock grows. One of the simplest ways of doing this is through the logistic model which is given by

\[ \frac{dP}{dt} = P \left( r - \frac{r}{K} P \right) \] (1)

The equation (1) is called the logistic equation. Its solution is known as the logistic function and its graph is known as the logistic curve. The model of population growth embodied in the logistic equation is called the logistic model. Here \( r \) is the ‘instantaneous’ growth rate when stock, \( P \), is very low, and \( K \) is the ‘carrying capacity’, the maximum stocks the environment is capable of supporting. This is a differential equation which can be solved explicitly.

The equation can be split up to give

\[ \frac{dP}{dt} = rP - \frac{rp}{K} P \]

The first part on the right hand side, \( rP \), is the same as the exponential function. The second part gives the reduction in growth rate as stock rises. When \( P = K \), stock is at the carrying capacity, and, as the equation shows, the growth rate is zero.

In ecological terms, the equation can be considered as consisting of a ‘density independent’ rate \( r \) and a ‘density dependent’ rate \( -\frac{rp}{K} \). One way to think of this is in terms of a rate at
which individuals grow when there is no competition and plentiful resources, being reduced by a rate reflecting competition or some other negative interaction as population increases towards the carrying capacity.

The usage of nonlinear differential equations is employed. Let \( P(t) \) be the size of a population at time, \( t \) and \( k > 0 \) is a constant [2]. Then the specific rate is defined by

\[
\frac{dP}{dt} = \frac{P}{P} \quad \text{(2)}
\]

It is assumed that the rate at which a population grows or declines is dependent only on the current number. It would not be dependent on any time-dependent system such as situations bordering on seasons. This can then be stated as

\[
\frac{dP}{dt} = P f(P) \quad \text{(3)}
\]

The equation (3) is called the density-dependent hypothesis [2]. With some assumptions equation (3) becomes (1). It is assumed that \( f(P) \) is linear and so \( f(0) = r \) and \( f(K) = 0 \).

Let \( \frac{r}{K} = b \), then the equation (1) becomes

\[
\frac{dP}{dt} = P(r - bP), \; b > 0 \; \text{and} \; r > 0. \quad \text{(4)}
\]

The solution of (4) is bounded as \( t \to \infty \).

**METHOD OF SOLVING THE LOGISTIC EQUATION**

Separation of variables is used to solve (4). Thus

\[
\frac{dP}{P(r-bP)} = dt
\]
Use partial fractions on the LHS to get: \[ \frac{dP}{P(r-bP)} = \left( \frac{1}{P} + \frac{b}{r-bP} \right) dP. \]

Hence we want to solve \[ \left( \frac{1}{P} + \frac{b}{r-bP} \right) dP = dt. \]

Integrating both sides of the above equation, we get

\[ \frac{1}{r} \ln|P| - \frac{1}{r} \ln|r-bP| = t + c \]

\[ \ln \left| \frac{P}{r-bP} \right| = rt + rc \]

\[ \frac{P}{r-bP} = c_1 e^{rt} \]

From the preceding equation it follows that

\[ P(t) = \frac{rc_1 e^{rt}}{1 + bc_1 e^{rt}} = \frac{rc_1}{bc_1 + e^{-rt}} \]

If \( P(0) = P_0 \), \( P_0 \neq \frac{r}{b} \), \( c_1 = \frac{P_0}{(r-bP_0)} \).

Then

\[ P(t) = \frac{rP_0}{bP_0 + (r-bP_0)e^{-rt}} \]

**DATA COLLECTION**

The data was collected from both undergraduate and post-graduate students in KNUST from all the six (6) Colleges during the Second Semester of the 2011/2012 academic year. In all, five hundred (500) students comprising 218 females and 282 males responded to the questionnaires. The questionnaires were administered continuously until the number was
exhausted. On the fifth day of ascertaining those who attend lectures regularly, seventy-six (76) were found to attend lectures regularly. The observations were carried out for some days and the data were documented. The fifth day observation was used to predict the rest and compare with the calculated and observed data.

It was assumed that the rate at which the affinity for lecture attendance was developed was proportional to the number of students. All the five hundred (500) students were observed through administering questionnaires to individual students within the space of fifteen days (See Table 1).

Table 1: Observed values of students who regularly attend lectures

<table>
<thead>
<tr>
<th>t(days)</th>
<th>x (number observed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>76</td>
</tr>
<tr>
<td>6</td>
<td>149</td>
</tr>
<tr>
<td>7</td>
<td>257</td>
</tr>
<tr>
<td>8</td>
<td>348</td>
</tr>
<tr>
<td>9</td>
<td>430</td>
</tr>
<tr>
<td>10</td>
<td>466</td>
</tr>
<tr>
<td>11</td>
<td>483</td>
</tr>
<tr>
<td>12</td>
<td>483</td>
</tr>
<tr>
<td>13</td>
<td>488</td>
</tr>
<tr>
<td>14</td>
<td>494</td>
</tr>
<tr>
<td>15</td>
<td>495</td>
</tr>
</tbody>
</table>

Source: field survey, 2012
CALCULATION OF RESULTS

Let $x$ be number of students who regularly attend lectures.

The observation is that $x(5) = 76$ the initial-value problem below was solved

$$\frac{dx}{dt} = Kx(500 - x).$$

Considering the following values $r = 500k, b = k$, and $P_0 = 1$, thus from the logistic equation

$$x(t) = \frac{500K}{K + 499Ke^{-500Kt}} = \frac{500}{1 + 499e^{-500Kt}}$$

From $x(5) = 76$, $K$ was determined from

$$x(5) = \frac{500}{1 + 499e^{-500K}} = 76, \text{ where } t = 5.$$  

Then substituting $t = 5$ and making $K$ the subject

$$500 = 76(1 + 499e^{-2500K})$$

$$424 = 76 \times 499e^{-2500K}$$

$$\frac{106}{9481} = e^{-2500K}$$

$$0.01118025525 = e^{-2500K}$$

Taking ln of both sides, we have

$$2500K = 4.493605981$$

substituting $K$ into the logistic equation we have

$$x(t) = \frac{500}{1 + 499e^{-0.001797442392 \times 500t}}$$

$$\therefore x(t) = \frac{500}{1 + 499e^{-0.8987t}}$$

Now the other calculations from day (6) to day (15) are shown below:

$$\therefore x(6) = \frac{500}{1 + 499e^{-0.8987(6)}} = 153$$
\[ x(7) = \frac{500}{1 + 499e^{-0.8987(7)}} = 260 \]

\[ x(8) = \frac{500}{1 + 499e^{-0.8987(8)}} = 363 \]

\[ x(9) = \frac{500}{1 + 499e^{-0.8987(9)}} = 434 \]

\[ x(10) = \frac{500}{1 + 499e^{-0.8987(10)}} = 471 \]

\[ x(11) = \frac{500}{1 + 499e^{-0.8987(11)}} = 488 \]

\[ x(12) = \frac{500}{1 + 499e^{-0.8987(12)}} = 495 \]

\[ x(13) = \frac{500}{1 + 499e^{-0.8987(13)}} = 498 \]

\[ x(14) = \frac{500}{1 + 499e^{-0.8987(14)}} = 499 \]

\[ x(15) = \frac{500}{1 + 499e^{-0.8987(15)}} = 500 \]

The above results have been depicted in the table below
Table 2: Calculated values of students who regularly attend lectures

<table>
<thead>
<tr>
<th>$t$ (days)</th>
<th>$x$ (number calculated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>76</td>
</tr>
<tr>
<td>6</td>
<td>153</td>
</tr>
<tr>
<td>7</td>
<td>260</td>
</tr>
<tr>
<td>8</td>
<td>363</td>
</tr>
<tr>
<td>9</td>
<td>434</td>
</tr>
<tr>
<td>10</td>
<td>471</td>
</tr>
<tr>
<td>11</td>
<td>488</td>
</tr>
<tr>
<td>12</td>
<td>495</td>
</tr>
<tr>
<td>13</td>
<td>498</td>
</tr>
<tr>
<td>14</td>
<td>499</td>
</tr>
<tr>
<td>15</td>
<td>500</td>
</tr>
</tbody>
</table>

Source: field survey, 2012
Comparing observed and calculated values. See Table 3

Table 3: Comparing observed and calculated values

<table>
<thead>
<tr>
<th>$t$ (days)</th>
<th>$X$ (number observed)</th>
<th>$X$ (number calculated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>76</td>
<td>76</td>
</tr>
<tr>
<td>6</td>
<td>149</td>
<td>153</td>
</tr>
<tr>
<td>7</td>
<td>257</td>
<td>260</td>
</tr>
<tr>
<td>8</td>
<td>348</td>
<td>363</td>
</tr>
<tr>
<td>9</td>
<td>430</td>
<td>434</td>
</tr>
<tr>
<td>10</td>
<td>466</td>
<td>471</td>
</tr>
<tr>
<td>11</td>
<td>483</td>
<td>488</td>
</tr>
<tr>
<td>12</td>
<td>483</td>
<td>495</td>
</tr>
<tr>
<td>13</td>
<td>488</td>
<td>498</td>
</tr>
<tr>
<td>14</td>
<td>494</td>
<td>499</td>
</tr>
<tr>
<td>15</td>
<td>495</td>
<td>500</td>
</tr>
</tbody>
</table>

Source: field survey, 2012
ANALYSIS OF RESULTS AND DISCUSSION

From Table 3, it is clear that the observed values and the calculated or predicted values are almost the same which render the model as a very good one. The seemingly vast differences noted occurred in day 8 with 348 as observed number of students whilst the calculated or predicted value came up to 363, day 12 with 483 as observed number of students whilst the calculated or predicted value came up to 495 and day 13 with 488 as observed number of students whilst the calculated or predicted value increased to 498. Apart from that the rest of the prediction were good.

Theorem

If \( f \) is a continuous real-valued function whose domain \( D(f) \) is an interval, then

\[
\left( \inf_{x \in D(f)} f(x), \sup_{x \in D(f)} f(x) \right) \subseteq R(f), \text{ where } R(f) \text{ denotes the range of } f.
\]

Determination of the range of \( x \)

Let \( x : [0, \infty) \to \mathbb{R} \) be defined by \( x(t) = \frac{500}{1 + 499e^{-0.8987t}} \).

\( x \) is continuous on \( [0, \infty) \)

\[
\inf_{x \in D(f)} x(t) = \inf_{t \in [0, \infty)} x(t) = \inf_{t \in [0, \infty)} \frac{500}{1 + 499e^{-0.8987t}} = \frac{500}{1 + 499e^{-0.8987(0)}} = 1
\]

\[
\sup_{x \in D(f)} x(t) = \sup_{t \in [0, \infty)} x(t) = \sup_{t \in [0, \infty)} \frac{500}{1 + 499e^{-0.8987t}} = \frac{500}{1 + 499e^{-0.8987(\infty)}} = 500
\]

Since \( t \to \infty, x(t) \to 500 \) hence the range of \( x(t) = [1, 500) \).
The graphs in Figures 1 and 2 which were obtained by the usage of matlab are shown below.

Figure 1: *Graph of $x$ (calculated number) against $t$ (days)*

The graph shows the calculated number of students' affinity for lecture attendance over time.
From Tables 1, 2, 3 and the graphs in Figures 1 and 2, it was noticed that the observed values and the calculated or predicted values almost converged to the same values.

CONCLUSION

A logistic differential equation had been used to model students’ affinity for lecture attendance in Kwame Nkrumah University of Science and Technology, Kumasi. The calculated or predicted values of students who regularly attended lectures were obtained by using logistic differential equation. The predicted values were then compared with the observed values and analyzed. From tables 1, 2, 3 and the graphs in figure 1 and 2 which were obtained by the usage of matlab, it was seen that the observed values and the predicted values almost converged to the same values, signifying that the model is good for the prediction of students’ preference for lecture attendance.
The study was also to gain insight into students’ reasons to attend or not to attend lectures in tertiary education environment in KNUST. Students were provided with a list of reasons for attending lectures and stated whether they applied to them or not. The result to this question indicates that, the reason that drive the majority of students to lectures are to find out what they are supposed to learn, not to miss important information and to find out about assessment tasks. However, trying to learn by oneself and expecting to be at lectures seem to be rarely encountered reasons. On the reasons why students do not attend lectures, majority of the respondents cited too long lecture periods, lectures integrated with tutorials, uninteresting courses and unexciting lecturing style usually make lectures boring.

Out of the 500 students who were interviewed in this study, 495 students were found to be students who attend lectures regularly and this shows affinity students of KNUST have for lecture attendance in spite of many internal and external factors that can have negative influence on lecture attendance.

The KNUST students’ preference for lecture attendance buttresses the notion that the lecture remains at the centre of most universities’ approaches to teaching and learning despite the emphasis on quality and flexibility, and the introduction of new technologies. To this end, it is therefore necessary to encourage lecture attendance in universities and other tertiary institutions.

In spite of many internal and external factors that have negative influence on lecture attendance, our study found out that, most of the students were profoundly appreciative of the relentless efforts lecturers and other stakeholders of tertiary education put in to improve lecture attendance so as to encourage students to attend lectures regularly.
RECOMMENDATIONS

The following recommendations were made:

- The learning university quality control division should identify training needs and organize symposiums, workshops, conferences and staff development programmes for lecturers so as to enhance their skills and performance levels.
- Students should be encouraged not to consider on-line lecture notes as a substitutes for lectures but use them as complements for lectures.
- Lectures and tutorials should be offered separately so as to avoid long lecture periods.

REFERENCES


